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The focus of this research project has been the study of systems of conservation laws that lose strict hyperbolicity. Two areas of application, namely isentropic gas dynamics for a van der Waals gas, and multiphase flow in a porous medium were considered. Additionally, a numerical study of the elastic string equations using Glimm's random choice method has been completed.

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NONLINEAR HYPERBOLIC CONSERVATION LAWS

Final Technical Report

Michael Shearer

June 30, 1987

U.S. Army Research Office

Grant Number: DAAL03-86-K-0004

Department of Mathematics

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A. Statement of Work

The focus of this research project has been the study of systems of conservation laws that lose strict hyperbolicity. The approach has been to consider two areas of application, namely isentropic gas dynamics for a van der Waals gas, and multiphase flow in a porous medium. Additionally, a numerical study of the elastic string equations using Glimm's random choice method has been completed.

B. Summary of Results

1. I have considered the role of undercompressive shocks, or phase jumps, insolving the initial value problem for isentropic gas dynamics with a nonmonotone equations of state. The main results are:

(a) If the initial data is close to the Maxwell line of equilibrium thermodynamics, then the Riemann initial value problem has a centered solution consisting of a fast wave and a slow wave separated by a slow phase jump (see [1]).

(b) For certain initial data, the Riemann problem has two solutions, distinguished by the presence or absence of a pair of phase jumps. This is summarized in [2].

2. The equations of motion of three phase flow in a porous medium constitute a 2x2 system of conservation laws. In a recent paper [3], I used a topological argument to show that these equations are generally of mixed elliptic and hyperbolic type. The elliptic part of state space is a bounded region.

The main part of this project has been the study of equations with isolated umbilic points. These correspond to elliptic regions shrunk to a point. In [4], we classified equations with umbilic points by obtaining a normal form for equations with quadratic nonlinearities. The equations of porous media flow fall into two of the four classes, whereas all four classes have been found in applications in elasticity, in which there is additional symmetry. In [5], we solved the Riemann problem for one case having symmetry, introducing undercompressive shocks, and ideas from bifurcation theory. In [6], the bifurcation ideas are developed further to solve the Riemann problem in three of the four cases. This paper also introduces and classifies boundaries of regions in state space across which the solution of the Riemann problem undergoes a qualitative change such as a change in the type of waves in the solutions.

One objective of this research has been to understand the solution of the Riemann problem sufficiently well that an algorithm can be developed to automate the solution. This is now being implemented by computing the wave curves (themselves boundaries), which undergo transitions across boundaries. The classification of boundaries plays an important role in the computations.

3. The implementation of Glimm's scheme for the 4x4 system of first order equations describing the motion of an elastic string has been carried out in [7]. The algorithm uses the solution of the Riemann problem that I had worked out earlier in a form that is computationally viable. The boundary conditions are handled by solving the Goursat (quarter plane) problem, which has a subtlety due to the possibility of the string doubling over. The most significant part of this work is the observed link between new explicit solutions of the equations, involving purely transverse motion, and a numerical solution that appears to be

periodic, and almost entirely transverse. This is significant because it is generally thought that shocks form from spatially periodic initial data, but no theorems are known for 4x4 systems.

C. Publications 1985-1987:

1. Dynamic phase transitions for a van der Waals gas. Quarterly of Applied Mathematics. To appear.

2. Phase jumps near the Maxwell line. Contemporary Mathematics 60(1987).

3. Loss of strict hyperbolicity for the Buckley-Leverett equations of three phase flow in a porous medium. Proceedings IMA Workshop on Numerical Oil Reservoir Simulation (ed. M. Wheeler) to appear.

4. (with D.G. Schaeffer) The classification of 2x2 systems of nonstrictly hyperbolic conservation laws, with application to oil recovery. Comm. Pure Appl. Math. 40(1987), pp. 141-178.

5. (with D.G. Schaeffer, D. Marchesin and P. Paes-Leme) Solution of the Riemann problem for a prototype 2x2 system of nonstrictly hyperbolic conservation laws. Arch. Rat. Mech. Anal. 97(1987), pp. 299-320.

6. (with D.G. Schaeffer) Riemann problems for nonstrictly hyperbolic conservation laws. Trans. A.M.S. To appear.

7. (with J. Fehribach) Glimm's scheme for the equations of motion of elastic strings. To appear.

Final Summary of Expenditures

Michael Shearer

Contract Number: DAAL03-86-K-0004

Direct Costs:		
Salary		7,850.01
Fringe Benefits (13.8% of salary)		1,083.39
Publications		14.58
Supplies		600.00
Travel		1,425.60
	Subtotal	10,973.58
Equipment		2,568.05
Overhead		4,410.37
Total Expenditures		13,541.63
	Total	17,952.00

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